

# Renormalization Group Functions for the Radiative Symmetry Breaking Scheme

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We obtain the renormalization group(RG) functions for the massless scalar field theory where symmetry breaking occurs radiatively. After obtaining the effective potential for the radiative symmetry breaking scheme from that of the minimal subtraction(MS) scheme by finite transformations for the classical field and coupling constant, we calculate the corresponding change of the RG functions.

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In the radiative symmetry breaking scheme[1], the scalar field does not have a tree level mass and the spontaneous symmetry breaking occurs radiatively from the effective potential[2]. The electroweak(EW) model where the symmetry breaking occurs radiatively have been studied extensively[3] due to its predictive power for the magnitude of the Higgs boson mass. In order to investigate the higher-order perturbative corrections in case of the large Higgs self-coupling, we need the RG functions for the radiative symmetry breaking scheme. Recently, it was conjectured[4] that the RG functions for the radiative symmetry breaking scheme ( $\beta_R$ ) can be obtained from that of the minimal subtraction scheme ( $\beta_{MS}$ ) as

$$\beta_R(\lambda) \stackrel{?}{=} \frac{\beta_{MS}(\lambda)}{1 - \frac{\beta_{MS}(\lambda)}{2\lambda}} \quad (1)$$

and

$$\gamma_R(\lambda) \stackrel{?}{=} \frac{\gamma_{MS}(\lambda)}{1 - \frac{\beta_{MS}(\lambda)}{2\lambda}} \quad (2)$$

In this paper, we will investigate the relation between  $\beta_R$  and  $\beta_{MS}$  systematically. Starting from the effective potential for the MS scheme whose renormalization functions are known, we will obtain the effective potential for the radiative symmetry breaking scheme by finite transformations of the field and coupling constant. Then we will calculate corresponding change of the renormalization functions. For simplicity, we will consider the case of the  $\lambda\phi^4$  theory and we will see that the above conjecture need corrections if the loop orders are higher than three.

The classical Lagrangian of the massless  $\lambda\phi^4$  model is given by

$$L = \frac{1}{2}(\partial_\mu\phi_B)^2 + \frac{\lambda_B}{24}\phi_B^4. \quad (3)$$

where the bare field  $\phi_B$  and the bare coupling constant  $\lambda_B$ . The effective potential of the massless  $\lambda\phi^4$  in the MS scheme has the form

$$V_{MS}(\lambda, \phi, \mu) = \sum_{l=0}^{\infty} \kappa^l \lambda^{l+1} \phi^4 \sum_{n=0}^l a_{l,n} L_{MS}^n \quad (4)$$

where  $\kappa = (16\pi^2)^{-1}$ ,  $a_{0,0} = \frac{1}{24}$ ,  $l$  is the order of the loop and

$$L_{MS} \equiv \log \left( \frac{\lambda\phi^2}{2\mu^2} \right). \quad (5)$$

Since the effective potential of the massless  $\lambda\phi^4$  model is independent of the renormalization mass scale  $\mu$ , it satisfies the renormalization group equation

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta_{MS}(\lambda) \frac{\partial}{\partial \lambda} + \gamma_{MS}(\lambda) \phi \frac{\partial}{\partial \phi} \right] V_{MS}(\lambda, \phi, \mu) = 0 \quad (6)$$

where the coefficients of the RG functions  $\beta_{MS}$  and  $\gamma_{MS}$  are given by[5]

$$\beta_{MS}(\lambda) = \mu \frac{d\lambda}{d\mu} = \sum_{l=1}^{\infty} \kappa^l \lambda^{l+1} b_l = 3\kappa\lambda^2 - \frac{17}{3}\kappa^2\lambda^3 + \left(\frac{145}{8} + 12 \varsigma(3)\right)\kappa^3\lambda^4 + \cdots \quad (7)$$

and

$$\gamma_{MS}(\lambda) = \frac{\mu}{\phi} \frac{d\phi}{d\mu} = \sum_{l=2}^{\infty} \kappa^l \lambda^{l+1} g_l = -\frac{1}{12} \kappa^2 \lambda^2 + \frac{1}{16} \kappa^3 \lambda^3 + \dots \quad (8)$$

The effective potential for the  $\lambda\phi^4$  theory in the MS scheme was obtained up to two-loop order[6]. In case of the three-loop order, we can determine terms which depend on logarithms by using the RG improvement of the effective potential[7]. As a result, we obtain

$$\begin{aligned} V_{MS}(\lambda, \phi, \mu) = & \phi^4 \left[ \frac{1}{24} \lambda + \kappa \lambda^2 \left( \frac{1}{16} L_{MS} - \frac{3}{32} \right) + \kappa^2 \lambda^3 \left( \frac{3}{32} L_{MS}^2 - \frac{5}{16} L_{MS} + \frac{11}{32} + \frac{1}{2} \Omega(1) \right) \right. \\ & \left. + \kappa^3 \lambda^4 \left( \frac{9}{64} L_{MS}^3 - \frac{143}{192} L_{MS}^2 + \left( \frac{701}{384} + \frac{\zeta(3)}{4} + \frac{9}{4} \Omega(1) \right) L_{MS} + a_{30} \right) + \dots \right] \end{aligned} \quad (9)$$

where

$$\Omega(1) = -\frac{1}{2\sqrt{3}} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin\left(\frac{n\pi}{3}\right) \simeq -0.293. \quad (10)$$

and  $a_{30}$  is a constant. It is easy to see that  $V_{MS}(\lambda, \phi, \mu)$  given in (9) satisfies the RG equation (6) up to the order  $O(\kappa^3)$  and  $a_{30}$  do not contribute up to this order.

### A. Two Step Procedure

In this procedure, we will follow two steps to transform the effective potential for the minimal subtraction  $V_{MS}(\lambda, \phi, \mu)$  into the effective potential for the radiative symmetry breaking and will investigate the corresponding change of the RG functions.

STEP I: In this step, by making a transformation  $\mu^2 \rightarrow \mu^2 \lambda/2$ , we change the logarithms  $L_{MS}$  in  $V_{MS}(\lambda, \phi, \mu)$  to  $\tilde{L}$  where  $\tilde{L} \equiv \log\left(\frac{\phi^2}{\mu^2}\right)$ . As a result, we obtain  $\tilde{V}(\lambda, \phi, \mu)$  where

$$\begin{aligned} \tilde{V}(\lambda, \phi, \mu) = & \sum_{l=0}^{\infty} \kappa^l \lambda^{l+1} \phi^4 \sum_{n=0}^l a_{l,n} \tilde{L}^n \\ = & \phi^4 \left[ \frac{1}{24} \lambda + \kappa \lambda^2 \left( \frac{1}{16} \tilde{L} - \frac{3}{32} \right) + \kappa^2 \lambda^3 \left( \frac{3}{32} \tilde{L}^2 - \frac{5}{16} \tilde{L} + \frac{11}{32} + \frac{1}{2} \Omega(1) \right) \right. \\ & \left. + \kappa^3 \lambda^4 \left( \frac{9}{64} \tilde{L}^3 - \frac{143}{192} \tilde{L}^2 + \left( \frac{701}{384} + \frac{\zeta(3)}{4} + \frac{9}{4} \Omega(1) \right) \tilde{L} + a_{30} \right) + \dots \right] \end{aligned} \quad (11)$$

Let us emphasize the fact that both  $V_{MS}(\lambda, \phi, \mu)$  and  $\tilde{V}(\lambda, \phi, \mu)$  contains the same coefficient  $a_{l,n}$ . It is known that[8] the  $\tilde{V}(\lambda, \phi, \mu)$  satisfies the RG equation

$$\left[ \mu \frac{\partial}{\partial \mu} + \tilde{\beta} \frac{\partial}{\partial \lambda} + \tilde{\gamma} \phi \frac{\partial}{\partial \phi} \right] \tilde{V}(\lambda, \phi, \mu) = 0 \quad (12)$$

where

$$\tilde{\beta}(\lambda) = \frac{\beta_{MS}(\lambda)}{1 - \frac{\beta_{MS}(\lambda)}{2\lambda}} = \sum_{l=1}^{\infty} \kappa^l \lambda^{l+1} \tilde{g}_l \quad (13)$$

and

$$\tilde{\gamma}(\lambda) = \frac{\gamma_{MS}(\lambda)}{1 - \frac{\beta_{MS}(\lambda)}{2\lambda}} = \sum_{l=1}^{\infty} \kappa^l \lambda^{l+1} \tilde{b}_l \quad (14)$$

By expanding (13) and (14), we obtain  $\tilde{b}_l$  and  $\tilde{g}_l$ :

$$\tilde{b}_1 = b_1 = 3 \quad (15)$$

$$\tilde{b}_2 = b_2 + \frac{1}{2}b_1^2 = -\frac{7}{6} \quad (16)$$

$$\tilde{b}_3 = b_3 + b_1b_2 + \frac{1}{4}b_1^3 = \frac{63}{8} + 12\varsigma(3) \quad (17)$$

$$\tilde{g}_2 = g_2 = -\frac{1}{12} \quad (18)$$

$$\tilde{g}_3 = g_3 + \frac{1}{2}g_2b_1 = -\frac{1}{16} \quad (19)$$

Then one can easily see that  $\tilde{V}(\lambda, \phi, \mu)$  given in (11) satisfies the RG equation (12) up to the order  $O(\kappa^3)$ . In order to see that the the RG equation (12) is satisfied to all orders, let us first substitute  $V_{MS}(\lambda, \phi, \mu)$  given in (4) to the RG equation (6). As a result, we obtain

$$\sum_{l=0}^{\infty} \kappa^l \lambda^{l+1} \phi^4 \sum_{n=1}^l [-2na_{l,n} + \frac{\beta_{MS}(\lambda)}{\lambda} \{(l+1)a_{l,n-1} + na_{l,n}\} + \gamma_{MS}\{4a_{l,n-1} + 2na_{l,n}\}] L_{MS}^{n-1} = 0 \quad (20)$$

If we substitute the perturbative expansions given in Eqs.(7) and (8) into the above equation, we can determine the coefficients  $a_{l,n}$ . Since the  $L_{MS}$  in (20) is independent of  $a_{l,n}$ , we can replace it with any quantity which is independent of  $a_{l,n}$ . Then, by replacing  $L_{MS}$  with  $\tilde{L}$  in (20) and by arranging terms, we obtain

$$\sum_{l=0}^{\infty} \kappa^l \lambda^{l+1} \phi^4 \sum_{n=1}^l [-(1 - \frac{\beta_{MS}(\lambda)}{2\lambda})2na_{l,n} + \frac{\beta_{MS}(\lambda)}{\lambda}(l+1)a_{l,n-1} + \gamma_{MS}\{4a_{l,n-1} + 2na_{l,n}\}]\tilde{L}^{n-1} = 0 \quad (21)$$

By using (11), we can write above equation as

$$[(1 - \frac{\beta_{MS}(\lambda)}{2\lambda})\mu \frac{\partial}{\partial \mu} + \beta_{MS}(\lambda) \frac{\partial}{\partial \lambda} + \gamma_{MS}(\lambda) \phi \frac{\partial}{\partial \psi}] \tilde{V}(\lambda, \phi, \mu) = 0. \quad (22)$$

and by dividing this equation with  $(1 - \frac{\beta_{MS}(\lambda)}{2\lambda})$ , we obtain (12).

STEP II: In order to transform the  $\tilde{V}(\lambda, \phi, \mu)$  into the effective potential for the radiative symmetry breaking scheme  $V_R$ , we need to add finite counterterms to satisfy given renormalization condition. As in case of the infinite counterterms, these finite counterterms will also contribute to the higher order loop expansion and this amounts to the redefinition of the coupling constants[9]. Then, by defining new coupling constant  $\eta$  and the classical field  $\psi$  as

$$\lambda(\eta) = \sum_{l=0}^{\infty} \kappa^l c_l(\eta) \quad (c_0 = 1) \quad (23)$$

and

$$\phi(\psi, \eta) = \psi \sum_{l=0}^{\infty} \kappa^l d_l(\eta) \quad (d_0 = 1) \quad (24)$$

we can obtain the effective potential for the radiative symmetry breaking  $V_R(\eta, \psi, \mu)$  as

$$V_R(\eta, \psi, \mu) = \tilde{V}(\lambda(\eta), \phi(\psi, \eta), \mu) \quad (25)$$

which should satisfy the given renormalization condition

$$\left[ \frac{d^4 V_R(\eta, \psi, \mu)}{d\psi^4} \right]_{\psi=\mu} = \eta. \quad (26)$$

The constants  $c_l$  and  $d_l$  can be determined order by order from the renormalization condition given in (25). Actually, we obtain

$$c_1 = -4\eta^2, \quad d_1 = 0 \quad (27)$$

$$c_2 = (\frac{115}{4} - 12\Omega(1))\eta^3, \quad d_2 = 0 \quad (28)$$

$$c_3 = (-\frac{28465}{96} + 15\Omega(1) - 25\varsigma(3) - 24a_{30})\eta^4, \quad d_3 = 0. \quad (29)$$

Althouh  $d_i = 0$  in this step, it turns out that  $d_i \neq 0$  in case of the one step procedure (see(40) and (41)). Then  $\beta_R(\eta)$  can be obtained as

$$\beta_R(\eta) \equiv \mu \frac{\partial \eta}{\partial \mu} = \frac{\partial \eta}{\partial \lambda} \mu \frac{\partial \lambda}{\partial \mu} = \frac{\tilde{\beta}(\lambda(\eta))}{\frac{\partial \lambda(\eta)}{\partial \eta}}. \quad (30)$$

In order to obtain  $\gamma_R(\eta)$ , let us invert the Eq.(24) as

$$\psi(\phi, \eta) = \phi[1 - \kappa d_1 + \kappa^2(d_1^2 - d_2) + \kappa^3(-d_3 + 2d_2d_1 - d_1^3) \cdot \dots] \equiv \phi \sum_{l=0}^{\infty} \kappa^l e_l(\eta). \quad (31)$$

Then  $\gamma_R(\eta)$  can be obtained as

$$\begin{aligned} \gamma_R(\eta) &\equiv \frac{\mu}{\psi} \frac{\partial \psi}{\partial \mu} = \frac{1}{\psi} \left[ \mu \frac{\partial \phi}{\partial \mu} \sum_{l=0}^{\infty} \kappa^l e_l(\eta) + \phi \mu \frac{\partial \eta}{\partial \mu} \sum_{l=0}^{\infty} \kappa^l \frac{\partial e_l(\eta)}{\partial \eta} \right] \\ &= \tilde{\gamma}(\lambda(\eta)) + \beta_R(\eta) \sum_{l=0}^{\infty} \kappa^l d_l(\eta) \sum_{m=0}^{\infty} \kappa^m \frac{\partial e_l(\eta)}{\partial \eta}. \end{aligned} \quad (32)$$

Note that the coefficients  $c_l$  and  $d_l$  determine the order  $\kappa^{l+1}$  terms of the RG functions. By substituting Eqs.(23),(24),(27),(28) and (29) to Eqs.(23),(24) and(25), we obtain the effective potential and the RG functions as

$$\begin{aligned} V_R(\eta, \psi, \mu) &= \phi^4 \left[ \frac{1}{24} \eta + \kappa \eta^2 \left( \frac{1}{16} L - \frac{25}{96} \right) + \kappa^2 \eta^3 \left( \frac{3}{32} L^2 - \frac{13}{16} L + \frac{55}{24} \right) \right. \\ &\quad \left. + \kappa^3 \eta^4 \left( \frac{9}{64} L^3 - \frac{359}{192} L^2 + \left( \frac{3905}{384} + \frac{\varsigma(3)}{4} + \frac{3}{4} \Omega(1) \right) L - \left( \frac{53845}{2304} + \frac{25}{8} \Omega(1) + \frac{25}{24} \varsigma(3) \right) \right) + \dots \right], \end{aligned} \quad (33)$$

where  $L \equiv \log \left( \frac{\psi^2}{\mu^2} \right)$

$$\begin{aligned} \beta_R(\eta) &= \kappa \tilde{b}_1 \eta^2 + \kappa^2 \tilde{b}_2 \eta^3 + \kappa^3 (\tilde{b}_3 + (c_1^2 - c_2) \tilde{b}_1 + c_1 \tilde{b}_2) \eta^4 + \dots \\ &= 3\kappa \eta^2 - \frac{7}{6} \kappa^2 \eta^3 + \left( -\frac{617}{24} + 12 \varsigma(3) + 36\Omega(1) \right) \kappa^3 \eta^4 + \dots \end{aligned} \quad (34)$$

and

$$\begin{aligned} \gamma_R(\eta) &= \tilde{g}_2 \kappa^2 \eta^2 + (\tilde{g}_3 + 2c_1 \tilde{g}_2) \kappa^3 \eta^3 + \dots \\ &= -\frac{1}{12} \kappa^2 \eta^2 + \frac{29}{48} \kappa^3 \eta^3 + \dots \end{aligned} \quad (35)$$

Note that  $V_R(\eta, \psi, \mu)$  agrees with the N=1 case of the two loop effective potential for the O(N) symmetric scalar field theory[10] and satisfies the RG equation

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta_R(\eta) \frac{\partial}{\partial \eta} + \gamma_R(\eta) \psi \frac{\partial}{\partial \psi} \right] V_R(\eta, \psi, \mu) = 0 \quad (36)$$

up to the order  $\kappa^3$ .

## B. One Step Procedure

We can obtain the effective potential for the radiative symmetry breaking scheme directly from the effective potential for the minimal subtraction  $V_{MS}(\lambda, \phi, \mu)$  without using STEP.I of the two step procedure by using the finite transformation of the coupling constant  $\lambda$  and the classical field  $\phi$  as given in Eqs.(23) and (24). Then the effective potential for the radiative symmetry breaking scheme  $V_R(\eta, \psi, \mu)$  defined by

$$V_R(\eta, \psi, \mu) = V_{MS}(\lambda(\eta), \phi(\psi, \eta), \mu) \quad (37)$$

should satisfy following conditions:

(i)  $V_R(\eta, \psi, \mu)$  should satisfy both the renormalization condition given in (26).

(ii) All the  $\log^m \left( \frac{\lambda \phi^2}{2\mu^2} \right)$  ( $1 \leq m \leq l$ ) terms in the order  $\kappa^l$  terms of  $V_{MS}$  should be transformed to  $\log^m \left( \frac{\psi^2}{\mu^2} \right)$ .

As a result of the condition (ii), the order  $\kappa^l$  term of the coefficients  $c_l$  and  $d_l$  contains the terms depending on powers of  $\log \left( \frac{\eta}{2} \right)$ . In the present case, the RG functions are given by

$$\beta_R(\eta) = \frac{\beta_{MS}(\lambda(\eta))}{\frac{\partial \lambda(\eta)}{\partial \eta}}. \quad (38)$$

and

$$\gamma_R(\eta) = \gamma_{MS}(\lambda(\eta)) + \beta_R(\eta) \sum_{l=0}^{\infty} \kappa^l d_l(\eta) \sum_{m=0}^{\infty} \kappa^m \frac{\partial e_l(\eta)}{\partial \eta}. \quad (39)$$

instead of (30) and (32). If we substitute the resulting coefficients to Eqs.(38) and (39), we can obtain the RG functions  $\beta_R(\eta)$  and  $\gamma_R(\eta)$ . It turns out that these two conditions does not fix the coefficients  $c_l$  and  $d_l$  uniquely and some choices can lead to RG functions depending on the  $\log \left( \frac{\eta}{2} \right)$  terms. In order to fix the coefficients  $c_l$  and  $d_l$  uniquely, we demand additional condition:

(iii) The resulting order  $\kappa^{l+1}$  terms of the RG functions does not contains the  $\log \left( \frac{\eta}{2} \right)$ .

Then, the coefficients  $c_l$  and  $d_l$  can be obtained as

$$c_1 = \left( -\frac{3}{2} \log \left( \frac{\eta}{2} \right) - 4 \right) \eta^2, \quad (40)$$

$$c_2 = \left( \frac{9}{4} \log^2 \left( \frac{\eta}{2} \right) + \frac{89}{6} \log \left( \frac{\eta}{2} \right) + \frac{139}{4} - 12\Omega(1) \right) \eta^3, \quad (41)$$

$$d_1 = 0 \quad (42)$$

$$d_2 = \frac{1}{24} \eta^2 \log \left( \frac{\eta}{2} \right). \quad (43)$$

By substituting the coefficients  $c_l$  and  $d_l$  into Eqs. (37),(38) and (39) we obtain the exactly same effective potential and RG functions as in Eqs.(33),(34) and (35).

In summary, we have obtained the RG functions for the radiative symmetry breaking scheme from those for the minimal subtraction scheme by two different ways. One was by changing the mass scale  $\mu$  followed by a finite transformations of the coupling constants and classical field to satisfy the renormalization condition. The other was by using only the finite transformations of the coupling constants and classical field that depends on logarithms of the coupling constants. We have confirmed that the two different methods give the same results for the effective potential and the RG functions and satisfy the RG equation up to three-loop order. The latter method can be used in case of the theories which have two or more different mass scales say in  $O(N)$  symmetric scalar field theory and scalar electrodynamics and this is in progress.

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- [1] S. Coleman and E.Weinberg, Phys. Rev. D7, 1888 (1973).
  - [2] For a review and references, see M. Sher, Phys. Rep. 89, 273 (1989).
  - [3] V. Elias, R.B.Mann, D.G.C.McKeon and T.G.Stelle, Phys. Rev. Lett. 91, 251601 (2003); Nucl. Phys. B678, 147 (2004); Phys. Rev. D72, 037902 (2005); F.A.Chishtie, V. Elias, R.B.Mann, D.G.C.McKeon and T.G.Stelle, Nucl. Phys. B743, 104 (2006).
  - [4] F.A.Chishtie, D.G.C.McKeon and T.G.Stelle, arXiv:0802.2919; F.A.Chishtie, T.Hanif, D.G.C.McKeon and T.G.Stelle, Phys. Rev. D77, 065007 (2008).
  - [5] H. Kleinert, J. Neu, V. Schulte-Frohlinde, K. G. Chetyrkin, S. A. Larin, Phys. Lett. B272, 39(1991); ibid. 319, 545(1993).
  - [6] C.Ford, D.R.T.Jones, P.W.Stephenson and M.B.Einhorn, Nucl. Phys. B395, 17(1993).
  - [7] B. Kastening, Phys. Lett. B283, 287 (1992); M. Bando, T. Kugo, N. Maekawa and H. Nakano, Phys. Lett. B301, 89 (1993).
  - [8] C. Ford, D. R. T. Jones, Phys. Lett. B274, 409 (1992), 285, 399(E) (1992).
  - [9] D. R. T. Jones, Nucl. Phys. B, 153 (1986).
  - [10] R.Jackiw, Phys.Rev. D9, 1686 (1974).